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ABSTRACT

No matter how well intentioned the Goals 2000: Educate America Act may be, it is doomed to failure. The theory of the logic and behavior of the educational system illustrates how powerful systemic forces converge to stabilize the high school attainment rate at about 75%, where it has been since 1965, and where no traditional national policy will be able to advance it very much. If education policy could increase the rate to 90% or beyond, undesirable consequences of potentially great magnitude, especially for targeted minority groups, would result. Economic disaster would result for those who could not complete or chose not to complete high school. Social benefits would also be reduced for those who did complete high school, and the diploma would lose its power to secure social goods. Part 2 of this paper outlines a theory of the logic and behavior of educational systems, and Part 3 presents a noncausal aggregate model that illustrates system dynamics. Part 4 presents an individual probabilistic model to extend the aggregate model, and Part 5 analyzes the two models and discusses policy alternatives. Part 6 analyzes two models from Raymond Moudon that support the results reported. Appendixes present mathematics behind the models and an educational attainment table. (Contains 12 figures, 6 tables, and 14 references.) (SLD)

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# National Education 'Goals 2000':
## Some Disastrous Unintended Consequences

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## "Goals 2000: Educate America Act"

**SEC. 102 NATIONAL EDUCATION GOALS.**

(2) SCHOOL COMPLETION. --(A) By the year 2000, the high school graduation rate will increase to at least 90 percent. (B) The objectives for this goal are that--

(i) the Nation must dramatically reduce its school dropout rate, and 75 percent of the students who do drop out will successfully complete a high school degree or its equivalent; and

(ii) the gap in high school graduation rates between American students from minority backgrounds and their non-minority counterparts will be eliminated.

(Public Law 103-227, 1994)

## I. INTRODUCTION

The purpose of the Act is to promote "coherent, nationwide, systemic education reform." (Public Law 103-227, 20 USC 5801) However well intentioned such an attempt at reform may be, it is doomed to failure. Legislators and education policy makers ignore the laws and dynamics of the educational system at their own and our peril.

The "system of education" is a vast and complex enterprise comprising all of the many and different ways society educates it citizens. It is useful to distinguish it from the educational system which possesses a logic and laws of behavior of its own and which can be shown to be highly intractable to attempts to reform it by education policy. This is particularly true with regard to "Goals 2000: Educate America Act."

The theory of the logic and behavior of the educational system illustrates how powerful systemic forces converge to stabilize the high school attainment rate at about 75% where it has been since 1965 and where no traditional national education policy will be able to advance it very much. Even if education policy could succeed in increasing the rate to 90%, or beyond, undesirable consequences of potentially great magnitude, especially for the targeted minority groups, would result.

One undesirable consequence is economic disaster for those who cannot or choose not to complete high school. They will be shut out of important non-educational social benefits (e.g., good job opportunities) unless alternative routes are opened for them. Another consequence is the potential reduction of these very same social benefits for those who do complete high school. A third consequence manifests itself as an unintended, but cruel hoax perpetrated upon the very minorities the Act seeks to help. By virtue of their being the last identifiable group to attain the

high school diploma in proportion to their numbers in the age cohort, the high school diploma will not have the same power to secure social goods as it did with previous groups.

Part II presents a brief outline of a comprehensive and general theory of the logic and behavior of national educational systems (Green, 1980). Certain of its laws and resulting dynamics are exposed. Part III presents a non-causal *a priori* aggregate model that illustrates certain systemic dynamics.

Part IV presents an individual probabilistic utility model that extends the aggregate model. Both models illustrate systemic theory with respect to the Congressional Act and serve to locate critical stages in the growth of the educational system where education policy is most and least effective.

Part V draws conclusions from the analyses of the two models and discusses several education and non-educational policy alternatives. Part VI is an analysis of the results of two models from Raymond Boudon which support the results reported here.

Appendices A and B contain the mathematics of the Individual Utility model. Appendix C contains the mathematics behind the Aggregate model. Appendix D contains an educational attainment table.[1]

## II. THEORY OF THE LOGIC AND BEHAVIOR OF THE EDUCATIONAL SYSTEM

A student who leaves school in the middle of the school year in one part of the country and who enters the same grade in a distant part of the country can generally find nearly identical curricula, procedures and facilities. It appears that some sort of system exists.

Education policy is after all, policy for the educational system. But what is the educational system? What are its features? What are the laws of its behavior that set the system in motion? Answers to these questions can help us to assess the potential impact of the Congressional Act.

***Primary Features.*** The primary features of the educational system are threefold.

**(1)** The set of schools and colleges, but not all schools and colleges.

**(2)** These schools and colleges within the system are connected by a medium of exchange which includes those certificates, degrees, diplomas, and the like, that allow one to leave the Nth level of the system in one locality and enter the Nth level in another. They are all instruments by which activities carried out in one place can be recognized and "exchanged" for similar activities of a school or college in some other place.

Certain schools and colleges will fall outside of the educational system although they will be within the system of education. Certain proprietary schools may not have their transcripts and diplomas recognized or accepted at other schools that are within the system.

**(3)** The schools and colleges that make up the educational system and that are connected by a medium of exchange are arranged by a principle of sequence: the system of colleges and schools are organized into levels so that if a person has attained (i.e., completed) level N, then he or she has attained level N-1, but not necessarily level N+1.

This principle allows us to speak of persons progressing through the system and seems to be a necessary property of any educational system due in part to differing levels of skill accomplishment, knowledge acquisition and the cognitive development of individuals.

***Secondary Features.*** The system also has certain secondary or derivative elements. They are size, a system of control and a distributive function.

**Distribution.** Every society makes some sort of arrangements for the distribution of its goods (i.e., benefits). The educational system distributes educational goods such as knowledge, skills, and certain kinds of taste, amongst others. In addition to these goods, the system distributes their surrogates, or second-order educational goods such as grades, diplomas, certificates and the like.

The derivative element of "control" is less relevant for the present analysis than the others. It turns out that size is of central import since education policy that is effective for one stage of systemic growth may be wholly ineffective at another.

## Figure II-1. The Modes of Growth

1. The system may expand in response to increases in the school-age population either by increasing the number of units in the system, or by increasing the number of students in units of the system, or both.

2. **Growth in attainment.** The system may expand by increasing rates of attendance and survival.

3. Vertical expansion. The system may expand by adding levels either at the top or at the bottom.

4. Horizontal expansion. The system may expand by assuming responsibility for educational and social functions that are either new, that have been ignored, or that have been carried out by other institutions.

5. Differentiation. The system may expand either by differentiation of programs or institutions or both.

6. Growth in efficiency. The system may expand by intensification, that is, by attempting to do more in the same time or the same in less time.

7. The system may expand by extending the school year or the school day.

8. The system may expand by increasing the number of persons needed to staff it *independently* of the number of students and number of its units, the magnitude of the school-age population, rates of attendance, survival.

(Green, 1980, pg. 10)

**System Size.** The educational system has eight distinct ways that it can grow (Figure II-1). The present analysis focuses upon "growth in attainment" not only because this is what the Act addresses, but because this mode of growth plays a crucial role in the dynamics of the system which in turn dooms the goals of the Congressional Act to certain failure.[2]

There are, however, two more pieces to the system that need to be developed before we can address the notion of growth and size. One is a normative principle connecting the social system with the educational system and the other is the systemic Law of Zero Correlation that relates the strength of the normative principle to system size.

***Normative Principle.*** It is true that some persons, for whatever reason, will come to possess a larger share of educational goods than other persons. This may be due to ability (however it is defined within the system), tenacity, acuity of choice and any number of other reasons.

If non-educational social goods such as income, earnings opportunities and status are distributed by the socioeconomic system on the basis of the distribution by the educational system of educational goods (through the instrumentality of second-order educational goods), then there exists a normative principle that connects the educational and socioeconomic systems.

This normative principle can be rendered as those having a greater share of educational goods merit or deserve a greater share of non-educational social goods. See Figure II-2. The importance and power of this normative principle is, as we shall see, a function of the size of the educational system as measured by the rate of high school attainment. It varies over different stages of systemic growth.

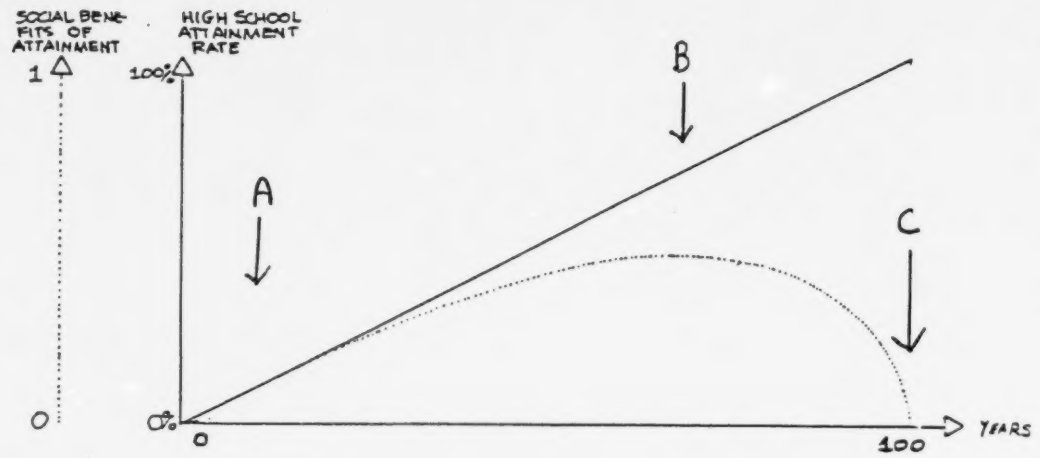
**Figure II-2. Normative Principle Components**

| Educationally Relevant Attributes | -----> | Educational Benefits | ------- > | Surrogate Benefits | ----------> | Non-educational Socioeconomic Benefits |
|---|---|---|---|---|---|---|
| ability, tenacity, acuity of choice, ... | | knowledge, skills, taste, manners, standards of civility, ... | | certificates, diplomas, transcripts, licenses, letters of recommendation, prestige, ... | | income, employment opportunities, status, ... |

***Law of Zero Correlation.*** To understand this law, let us posit a uniform growth curve. Suppose that the educational system grows at a uniform rate over a one hundred year period. That is, there is a uniform increase (10% each decade) in the proportion of each successive age-cohort attaining the 12th level of the system. (The actual growth data is shown in Appendix D.)

When the high school attainment rate is low (e.g., 10% ) the socioeconomic meaning of high school attainment is likely quite negligible. Employers, all things being equal, would have little reason to choose a high school graduate over a non-graduate especially when there are so many of the latter. In the aggregate, high school attainers do not monopolize economic opportunities simply because of attainment. Thus the strength of the normative principle is low. To be a high school drop out when most of your age-cohort drops out presents no serious personal or social problem. See Part A of Figure II-3.

**Figure II-3. Uniform Growth Curve & Social Benefits of Attainment**




_____ **High School Attainment Rate**
..... **Benefits of Attainment**

As the size of the educational system increases, the power of the normative principle also increases. Employers now utilize high school attainment as a selection criterion and social goods, such as status and jobs, begin to be preferentially distributed to high school graduates. See Part B of Figure II-3.

However, when the attainment rate reaches 100%, the mere possession of the high school diploma can have no socioeconomic meaning whatsoever. That is, no social goods can be distributed on the basis of high school attainment because everyone has the diploma. It is at this point (and at 0%), that the power of the normative principle is completely destroyed although its power may be weakened well before this point is reached. See Part C of Figure II-3.

The Law of Zero Correlation is a logical tautology. See Figure II-4. It is *a priori* true. For instance, a society could not distribute any of its goods based upon eye color if everyone had the same color eyes. The actual shape of this curve and its inflection points is an empirical matter. However, the models presented in this paper give us some guidance in locating the theoretical inflection points.



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## Figure II-4. The Law of Zero Correlation

There is a point in the growth of the system at which there is no longer any correlation between educational attainment and either the distribution of educationally relevant attributes in the population or the distribution of non-educational social goods associated with educational attainment.

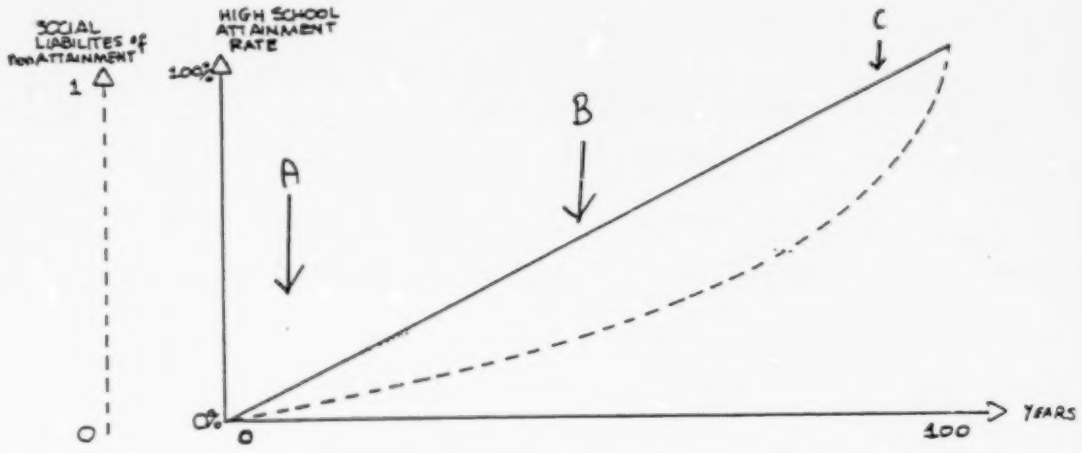
(Green, 1980, pg. 91)

***Law of Shifting Benefits and Liabilities.*** This is one of the many corollaries of the Law of Zero Correlation. This corollary assures that high school attainment will have a declining social value and that concomitantly, failure to attain the high school diploma will have an increasing social liability, as the attainment rate moves toward the 100% zero correlation point. Thus, as zero correlation is approached, the aggregate social benefits of the attainment group and the aggregate liabilities of non-attainment both increase (Figures II-3 and II-5)

On the liability side, where school leaving was once a possible and viable alternative, it now becomes an evil to be avoided at all costs. These shifting benefits and liabilities make high school attendance and attainment "compulsory" in ways that were surely never meant to be. The personal and social consequences of dropping out of high school can be devastating.

The Law of Shifting Benefits and Liabilities does not specify the points in systemic growth (Sections A, B and C in Figures II-3 and II-5) where the benefits and liabilities of high school attainment shift. However, the two models presented in Parts III and IV, do show that when 55% of the 17 year-old age-cohort attains the high school diploma, that group will receive the greater share of social benefits due to the moderate power of the normative principle.

**Figure II-5. Shifting Liabilities of Non-Attainment**



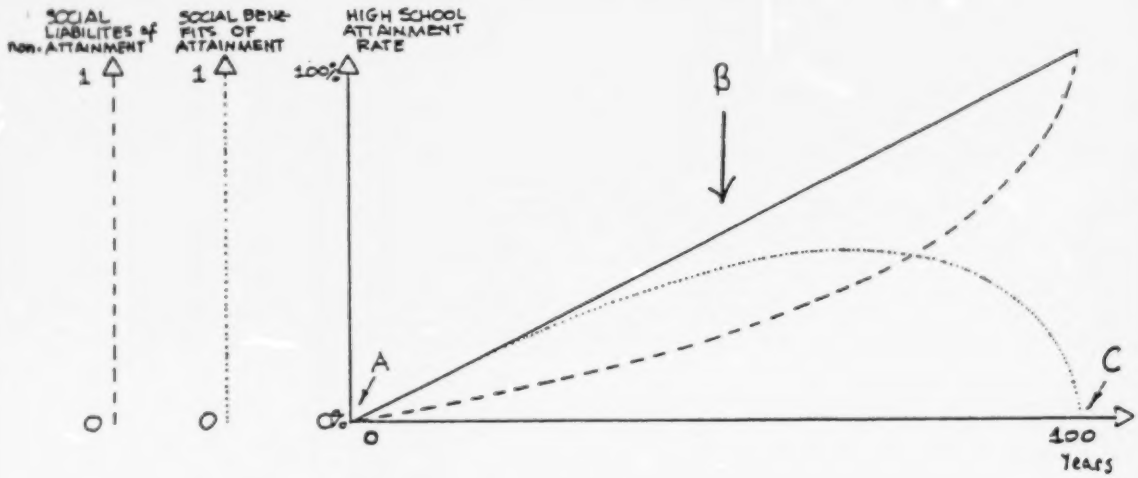

**High School Attainment Rate**
**Liabilities of Non-Attainment**

At this point in the growth of the educational system, high school attainment is efficacious in obtaining a disproportionate share of social goods. Thus, a high school diploma becomes a highly sought after good. This corresponds, in the growth of the system, to the year 1948.

In addition, the models show that when the system becomes fairly large (i.e., 76% high school attainment in 1965), the power of this normative principle begins to decrease even though, historically, the personal and social belief in it remains high. This is prior to zero correlation setting in and may explain why the system has stabilized at around 75% attainment and why it has been so resistant to attemps at education reform.

This is also the point at which the liabilities of non-attainment appear to increase dramatically and where the "drop out problem" became, politically, a problem to be dealt with. Figure II-6 shows the combined effects of the Law of Shifting Benefits and Liabilities and exposes a peculiar paradox: as zero correlation is approached, the aggregate social benefits once associated with high school attainment decline and the associated social liabilities of non-attainment increase.

**Figure II-6. Shifting Benefits and Liabilities of Attainment**




_____ **High School Attainment Rate**
..... **Benefits of Attainment**
---------- **Liabilities of Non-Attainment**

If one posits that Section C of Figure II-6 represents the part of the growth of the system where the effects of these laws are maximally felt, then what would befall the minorities that the Congressional Act seeks to help? To address this question, consider two more systemic principles: the Law of Last Entry and the Principle of the Moving Target. These two principles speak to the "Goals 2000" goal of closing the attainment gap (and presumably, the social benefits gap) between minorities and non-minority students.

The ***Law of Last Entry*** states that "as we approach the point of universal attainment at any level of the system, the last group to enter and complete that level will be drawn from lower socioeconomic groups." See Figure II-7. However, unlike the Law of Zero Correlation, this law is neither tautological nor *a priori*, but can be considered to be an empirical generalization. The basis for this claim is given in much more detail elsewhere (Green, 1980).

**Figure II-7. The Law of Last Entry**

It appears to be true that no society has been able to expand its total educational enterprise to include the lower status groups *in proportion to their numbers in the population* until the system is "saturated" by the upper and middle status groups.

(Green, 1980, 108)

A corollary of the Law of Last Entry is the ***Principle of the Moving Target***, which states that as the group of last entry reaches its target of proportional 12th grade attainment rate, the target will shift. Note, that if the group of last entry pushes the attainment rate to 100%, then the high school diploma cannot, in and of itself, be used to distribute social benefits to anyone, much less to this last group. Zero correlation will have set in and the target will have shifted to attaining a higher level of the educational system: post-secondary.

However, even if the attainment rate does not reach 100% with the group of last entry (in this case, minority groups), this group will still not reap the same benefits of the high school diploma that previous groups reaped due to the Law of Shifting Benefits and Liabilities. The point in the attainment growth where this occurs is an empirical point. However, the models presented in this paper give us some theoretical guidance.

"Goals 2000" seeks to set and carry out a national policy to increase the high school attainment rate from its present level to at least 90%. If the rate stays below 100 %, zero correlation would be avoided. I contend, however, that the effects of merely approaching zero correlation will be felt well before the 90% attainment level is reached (if it ever could be reached!). As the theoretical models which follow show, the felt effect could be one reason why the attainment rate has stabilized for so long at about 75%. Empirical confirmation can be found in (Green, 1980).

## III. THE AGGREGATE MODEL AND APPLICATIONS

### A. The Model

The following Aggregate Model rests upon three idealized assumptions:

1. Non-educational social benefits are always normally distributed in the population under consideration and remain so over time - a change in the high school attainment ratio does not affect the overall normal shape of this distribution;

2. This distribution encompasses those who have attained the high school diploma, but who have not gone on in formal schooling (attainers), and those who have not attained the high school diploma (non-attainers);

3. Society allocates its social benefits in such a way that the attainers monopolize the upper end of the normal distribution.

The first assumption fixes the overall shape of the distribution and offers a particular view of distributed justice. This distribution can be thought to reflect some overall normally distributed attribute or attributes in the total population under consideration. The second and third assumptions tell us that the high school attainers can be found, as a group, lumped at the upper end of the distribution. The third assumption, which admittedly represents an overly rigid meritocratic society, will be altered in the model presented in Part IV,

These three assumptions are realized in Figure III-2, which is a normal distribution in standardized normal form having a grand median ($\mu_o$) of zero and a standard deviation ($\sigma$) of one. Each asymptote is truncated, for computational purposes, at 3.9 standard deviations from the mean. The high school attainment ratio ($\Phi$) is represented by the shaded area under the curve. This is the proportion of the total population under consideration that has attained the high school diploma. The median value of the social benefits of this group is $\mu_\Phi$.

The unshaded portion under the curve is the proportion of the total population that has not attained the high school degree ($\underline{\Phi}$) and is equal to ($1 - \Phi$). The median value of the social benefit for this group is $\mu_{\underline{\Phi}}$.

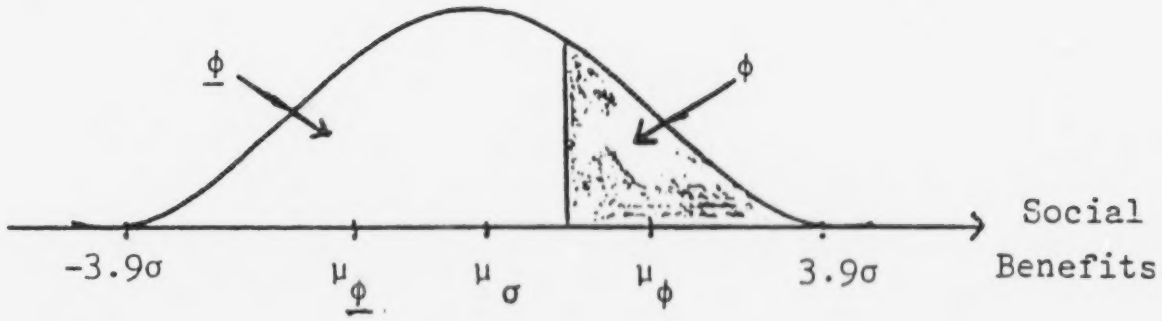



**Figure III-1. Standardized Normal Curve for the Distribution of Social Benefits**
($\Phi$ = high school attainment ratio; $\underline{\Phi}$ = non-attainment ratio;
$\mu_{\varphi} = 0$ = grand median; $\mu_{\Phi}$ = median social benefit for attainer group;
$\mu_{\underline{\Phi}}$ = median social benefit for non-attainer group;
$\sigma = 1$ = standard deviation)

**TABLE III-1**

**Median Social Benefits, Their Differences, and Their Rates of Change For Attainer and Non-attainer Groups by High School Attainment Ratio**

| (1) Size of Attainment Group ($\Phi$) | (2) Attainer Group Median ($\mu_{\Phi}$) | (3) Non-Attainer Group Median ($\mu_{\underline{\Phi}}$) | (4) $\mu_{\Phi} - \mu_{\underline{\Phi}}$ | (5) Rate of Change of $\mu_{\Phi}$ | (6) Rate of Change of $\mu_{\underline{\Phi}}$ |
|---|---|---|---|---|---|
| 0.01 | 2.575 | -0.012 | 2.587 | | |
| 0.05 | 1.960 | -0.063 | 2.023 | 0.2388 | 4.2500 |
| 0.10 | 1.645 | -0.126 | 1.771 | 0.1607 | 1.0000 |
| 0.15 | 1.440 | -0.189 | 1.629 | 0.1246 | 0.5000 |
| 0.20 | 1.283 | -0.253 | 1.536 | 0.1090 | 0.3386 |
| 0.25 | 1.150 | -0.319 | 1.469 | 0.1037 | 0.2609 |
| 0.30 | 1.037 | -0.385 | 1.422 | 0.0983 | 0.2069 |
| 0.35 | 0.935 | -0.454 | 1.389 | 0.0984 | 0.1792 |
| 0.40 | 0.842 | -0.524 | 1.366 | 0.0995 | 0.1542 |
| 0.45 | 0.755 | -0.598 | 1.353 | 0.1033 | 0.1412 |
| 0.50 | 0.675 | -0.675 | 1.350 | 0.1060 | 0.1288 |
| 0.55 | 0.598 | -0.755 | 1.353 | 0.1141 | 0.1185 |
| 0.60 | 0.524 | -0.842 | 1.366 | 0.1237 | 0.1152 |
| 0.65 | 0.454 | -0.935 | 1.389 | 0.1336 | 0.1105 |
| 0.70 | 0.385 | -1.037 | 1.422 | 0.1520 | 0.1091 |
| 0.75 | 0.319 | -1.150 | 1.469 | 0.1714 | 0.1090 |
| 0.80 | 0.253 | -1.283 | 1.536 | 0.2069 | 0.1157 |
| 0.85 | 0.189 | -1.440 | 1.629 | 0.2530 | 0.1224 |
| 0.90 | 0.126 | -1.645 | 1.771 | 0.3333 | 0.1424 |
| 0.95 | 0.063 | -1.960 | 2.023 | 0.5000 | 0.1915 |
| 0.99 | 0.012 | -2.575 | 2.587 | 0.8095 | 0.3138 |

Note that the attainer and non-attainer medians change as a function of the attainment ratio. When the ratio ($\Phi$) is zero, the non-attainer median is equal to the grand median ($\mu_o$). When the ratio approaches its limit of one, the attainer median approaches the grand median and the non-attainer median approaches -3.9 standard deviations from the grand median. We can easily calculate the values of the attainer and non-attainer medians for different values of the attainment ratio.[3] Table III-1 shows their values, their differences and their rates of change for attainment ratios ranging from 0.01 to 0.99. Figure III-2 is a plot of the attainer and non-attainer medians by the attainment ratio.

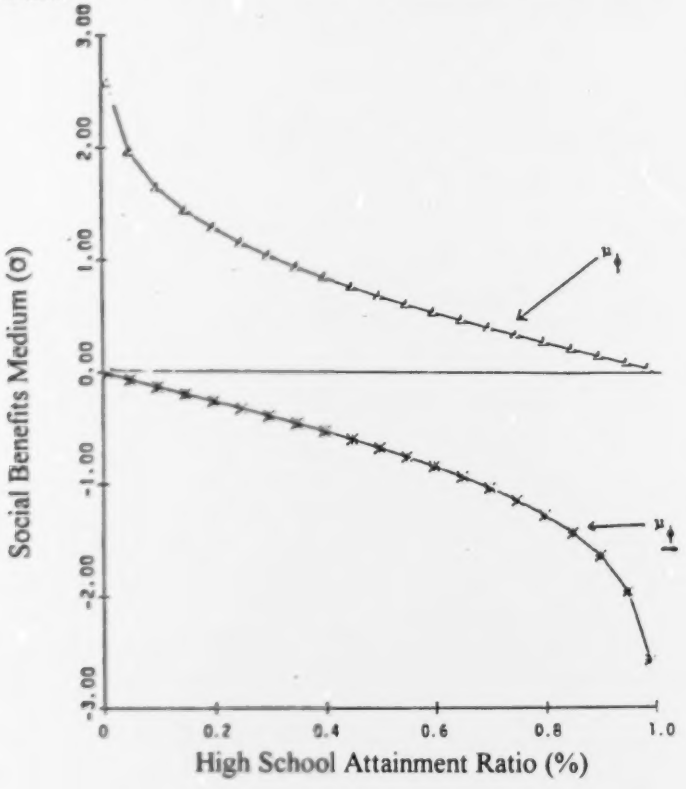



**Figure III-2** **Median Social Benefit of Attainer Group ($\mu_\Phi$) and Non-Attainer Group ($\mu_\Phi$) by High School Attainment Ratio ($\Phi$)**
(from Table III-1, Columns 2 and 3)

## B. An Income Disparity Analysis

A conventional analysis of high school attainer and non-attainer income disparities considers whatever is gained by the attainers to be the magnitude of the liability experienced by the non-attainers. If, for example, the median income of the attainer group is 150% of the non-attainer median income (at a particular attainment ratio), then the benefit to the former group is 50% while the liability to the latter group (in foregone income and earnings opportunities, etc.) is 50%. This approach tends to conceal the full impact of the shifting benefits and liabilities of educational attainment.

Table III-1 and Figure III-1 display another approach to this situation. Here we find the difference between the median benefit of the attainer group and the median benefit of the entire population under consideration (Table III-1, column 2). We do the same for the non-attainer group (Table III-1, column 3). The difference between these two grand-median-dispersions is a measure of the relative position of one group with respect to the other (Table III-1, column 4).

If we think of such social benefits as income, salary and wages, then a conventional supply and demand analysis suggests that as the supply of high school graduates increases, the relative social benefits realized by these graduates, with respect to those with no high school degree, will decline (given a constant market demand for attainers). This is just what happens in the Aggregate Model as the attainment ratio grows from 0 01 to 0.50. However, as the attainment ratio exceeds 50%, the relative advantage of the attainers over the non-attainers increases.[4] See Figure III-2.

These latter results of the model are consistent with certain empirical findings. Time-series U.S. Census data for 18-year-old to 24-year-old males from 1939 (when the national high school attainment ratio was 50%) to 1990 display this phenomenon.[5] A U.S. Senate report which examined the incomes of 24- to 34-year-old males expressed surprise at the "paradox" of increasing relative income for high school attainers over non-attainers.[6]

The interaction between the Law of Zero Correlation and the Law of Shifting Benefits and Liabilities has certain explanatory power when the data are examined as illustrated in the Aggregate Model. The "paradox," cited above, evaporates in light of these systemic dynamics which show the declining benefits associated with attainment and the increasing liabilities associated with non-attainment as the zero correlation point is approached.[7]

## C. Stabilization of the High School Attainment Ratio

What is the meaning of the "intersection" of the benefit and liability curves in Figure II-6? Although the two curves do not actually intersect (they have different vertical axes), the "intersection" shown in Figure II-6 does illustrate certain interactive systemic effects. This "intersection" can be viewed as an equilibrium point in the growth of the system beyond which it no longer pays (in aggregate social benefit terms) to finish high school but is quite a serious social disaster not to do so. In a way, it is an aggregate recognition of the Law of Zero Correlation and the Law of Shifting Benefits and Liabilities. This phenomenon is illustrated by the Aggregate Model.

Figure III-3 is a plot of the rate of decline of the social benefits of attainment generated by the model. Note that after an attainment ratio of 0.20 the median value declines at a fairly constant rate until the high school attainment ratio reaches 50%. At this point in the growth of the educational system, the rate of decline increases and increases sharply at 75% attainment.

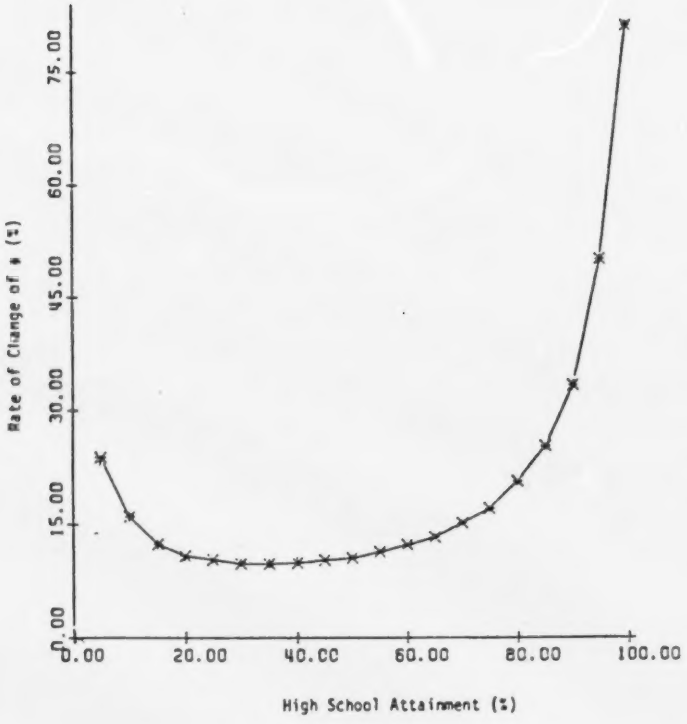



**Figure III-3. Rate of Change of Attainer Group Median by High School Attainment Ratio** (from Table III-1, Column 5)

Figure III-4 is a plot of the rate of decline of the non-attainer median. Here the median declines at a decreasing rate until 75% attainment at which point the rate begins to increase and then increases sharply at 80% attainment.

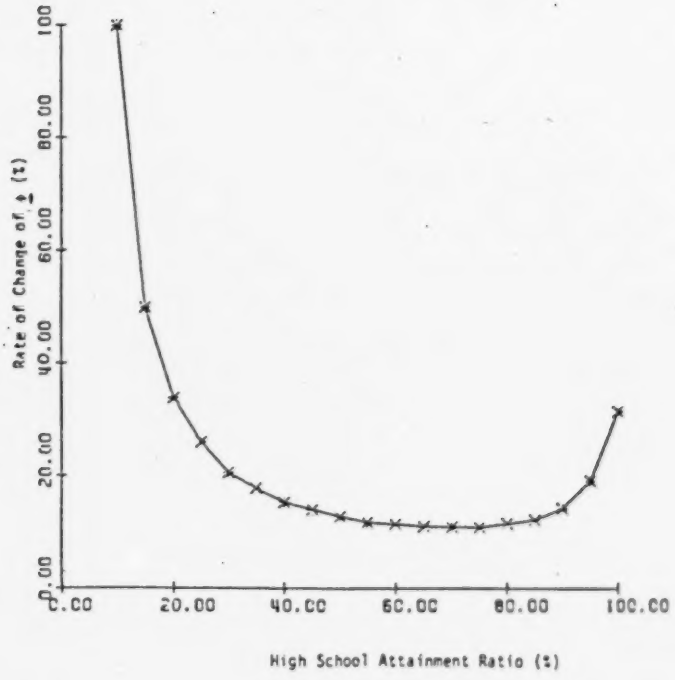



**Figure III-4** **Rate of Change of Non-Attainer Group Median by High School Attainment Ratio** (from Table III-1, Column 6)

Thus, the two curves shown in Figure III-2 can be said to contain inflection points which occur in the growth of the system where the high school attainment ratio is about 75%. The stabilization of the national attainment ratio at around 75% may be the social recognition of the phenomenon described by the model.[8]

Is it purely coincidental that the inflection points in the model occur where the national high school attainment ratio has stabilized at about 75%? Nevertheless, the model does serve to illustrate the phenomenon of systemic "equilibrium" reflecting the interactive dynamics between certain systemic laws. The interaction between these laws offers an account of certain systemic phenomena.

The behavior of the educational system described above is based upon these systemic features: the Principle of Sequence, the distribution of second-order educational goods and the size of the system as measured by the attainment ratio at the twelfth level. Systemic behavior was driven by the power of a logical tautology, its corollary and a normative principle linking the educational and social systems. It is ironic that the "successful" growth of the system, as measured by an increasing high school attainment ratio, appears to sow the seeds of an ironically harsh and peculiar brand of failure.[9]

## IV. THE INDIVIDUAL (PROBABILISTIC) UTILITY MODEL

The idealized society reflected in the three assumptions underlying the Aggregate Model is a rigidly meritocratic one. By altering the first and third assumptions, we can build a model that reflects a society that distributes its non-educational social goods in a somewhat more flexible manner.

Like the Aggregate Model, let us assume that the population under consideration is dichotomized into those who have attained the high school diploma (and nothing beyond it) and those who have not attained the degree. Furthermore, let us assume two independent normal distributions of social goods, one for the attainment group and the other for the non-attainment group. This state of affairs is illustrated in Figure IV-1.

Now let us assume that both of these normal distributions have identical standard deviations. Thus, we can normalize each of the distributions and leave them superimposed, one upon the other, on the social benefits axis. Note that the relative position of the two normal curve means remains unaffected by the standardization (i.e., the standardized and unstandardized means remain stationary). These standardized distributions are shown in Figure IV-1.

### A. The Standardized Normal Distributions

Consider the two standardized normal distributions shown in Figure IV-1. Let curves $X_{\phi}$ and $X_{\bar{\phi}}$ represent the distributions of earnings opportunities of high school attainers and non-attainers, respectively. Both curves have their asymptotes truncated, to facilitate the computations to follow, at 3.0 standard deviations above and below their respective means of zero and are superimposed upon a common axis, X, showing an apparent overlap area, E: that area under both curves which has a common X-axis range.

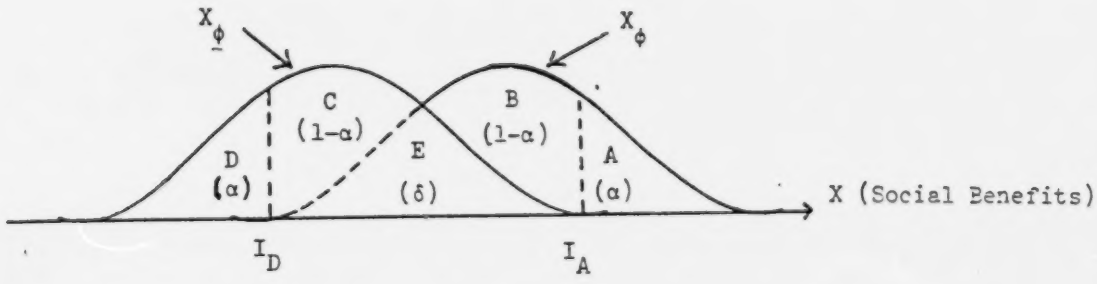



**Figure IV-1. Two Overlapping Standardized Normal Curves**

We let $\Phi$ stand for the ratio of high school attainers to the total population under consideration and let $\alpha$ stand for the meritocratic parameter. This parameter represents those in the total population, and in particular that proportion of distribution $X_{\Phi}$, which monopolizes the highest values of X. It is clear from Figure IV-1 that this parameter imposes an upper-bound on the range of distribution $X_{\underline{\Phi}}$ (i.e., $I_A$) and concomitantly places a lower-bound on the range of $X_{\Phi}$ (i.e., $I_D$). Except where $\alpha = 0$, the ranges of $X_{\Phi}$ and $X_{\underline{\Phi}}$ differ.

Let us assume that despite changes in the size of $\Phi$, the original non-standardized normal distributions retain their normal shapes and continue to have identical standard deviations and unchanged means. The $X_{\Phi}$ mean remains forever fixed and thus for any given $\Phi$, only a change in $\alpha$ can shift the $X_{\underline{\Phi}}$ curve. A mean/medium analysis of these curves is presented in Appendix B.

Unlike the Aggregate Model, individuals in $X_{\Phi}$ (i.e., high school attainers) are no longer guaranteed an advantage over persons in $X_{\underline{\Phi}}$ (i.e., non-attainers), with respect to some value of X (level of social benefit). The question now shifts from one of absolute advantage (as in the Aggregate Model) to one of relative advantage. We now ask, what is the probability that an individual will be advantaged with respect to X, over changes in $\Phi$ and in $\alpha$?

The symbols in Figure IV-1 refer to proportions and are explained in Table IV-1, below.

**TABLE IV-1**

**PROPORTIONAL VALUES OF SECTIONS IN FIGURE IV-1**

| Section | Symbol | Meaning |
|---|---|---|
| A | ($\alpha$) | The proportion of the population which is in $X_{\Phi}$ and which monopolizes the highest X values. This is the value of the meritocratic parameter. |
| B | ($1-\alpha$) | The proportion of the population which is in $X_{\Phi}$ and which does not monopolize the highest X values. |
| C | ($1-\alpha$) | The proportion of the population which is in $X_{\underline{\Phi}}$ and which is not relegated to the lowest X values. |
| D | ($\alpha$) | The proportion of the population which is in $X_{\underline{\Phi}}$ and is relegated to the lowest X values. |
| E | ($\delta$) | The area of "intersection" of Section B of $X_{\Phi}$ and Section C of $X_{\underline{\Phi}}$. |

The above conceptualization allows us to calculate the probabilities of persons falling in any of the five sections of Figure IV-1 as a function of $\alpha$ and $\Phi$. These probabilities are conditional probabilities of independent events. Table IV-2 gives the formulae for these calculations.

**TABLE IV-2**

**PROBABILITIES**

| Section | Probability | Meaning |
|---|---|---|
| A | $Pr(A\|X_\Phi)=\alpha\Phi$ | The probability of residing in Section A is the conditional probability of residing in A (i.e., $\alpha$) given that one already resides in $X_\Phi$ (i.e., $\Phi$). |
| B | $Pr(B\|X_\Phi)=(1-\alpha)\Phi$ | The probability of residing in Section B is the conditional probability of <u>not</u> residing in A (i.e., $1-\alpha$) given that one resides in $X_\Phi$ (i.e., $\Phi$). |
| C | $Pr(C\|X_\Phi)=(1-\alpha)(1-\Phi)$ | The probability of residing in Section C is the conditional probability of <u>not</u> residing in D (i.e., $1-\alpha$) given that one resides in $X_\Phi$ (i.e., $1-\Phi$). |
| D | $Pr(D\|X_\Phi)=\alpha(1-\Phi)$ | The probability of residing in Section D is the conditional probability of residing in D (i.e., $\alpha$) given that one resides in $X_\Phi$ (i.e., $1-\Phi$). |
| $E_1$ | $Pr(E\|C\|X_\Phi)=(1-\alpha)(1-\Phi)\delta$ | The probability of residing in Section E given that one is already in $X_\Phi$, is the conditional probability of residing in E (i.e., $\delta$) given that one resides in $X_\Phi$ (i.e., $1-\Phi$) <u>and</u> resides in Section C (i.e., $1-\alpha$). |
| $E_2$ | $Pr(E\|B\|X_\Phi)=(1-\alpha)\Phi\delta$ | The probability of residing in Section E given that one is already in $X_\Phi$ is the conditional probability of residing in E (i.e., $\delta$) given that one resides in $X_\Phi$ <u>and</u> resides in B (i.e., $1-\alpha$). |

**B. Interpretation of Area E**

The move from proportions in Table IV-1 to probabilities in Table IV-2 is a crucial one. Recall that each distribution represents one part of the dichotomized total population under consideration. The overlapping area E, is not a shared population between the two groups. It simply illustrates the common range of X shared by area B in $X_\Phi$ and C in $X_\Phi$.

Each person in the total population under consideration has a probability of ending up in one of the two distributions. Since $\Phi$ is the proportion of the total population that has attained the twelfth level, any individual has probability $\Phi$ of falling under distribution $X_\Phi$ (all other things being equal). Similarly, the probability of not attaining at level 12 is equal to $(1-\Phi)$. Of course, $\Phi + (1-\Phi)$ equals 1.0, which is the total population under consideration. All of this follows from the laws of proportions.

Consider Figure IV-1. As Section A changes in size, $X_\Phi$ shifts to the left or to the right (recall that we have assumed that changes in $\Phi$ do not affect the shape or position of the distributions). The entire area under any one of the two distributions is equal to 1.0. Thus, if $\alpha$ represents the value of the area of Section A, then $1-\alpha$ is the area of Section B. From this we can see that the conditional probability of an individual being an attainer and being a monopolizer of the higher values of X is $\alpha\Phi$.

The laws of symmetry make Section D equal to Section A. Thus, the probability of an individual being a non-attainer and being relegated to the lowest values of X is $\alpha(1-\Phi)$. Similar arguments can be made for Sections B and C. The probabilistic interpretation of Section E is a more complicated matter, however.

Although Sections B and C do not actually have an area in common, they do share the common X-axis range, $I_D$ to $I_A$. It is useful to think of Section E as if it is the area of overlap between the two distributions. Recall that the probability of being in C is simply $(1-\Phi)(1-\alpha)$. Now, the probability of being in C and at the same time being within the scope of distribution $X_\Phi$ is just the probability of being in C times the area of Section E. Similarly, the probability of being in B is $(1-\alpha)\Phi$. The probability of being in B and within the scope of distribution $X_\Phi$ is just the probability of being in B times the area of Section E.

It should now be clear that $Pr(E|C|X_\Phi)$ is the probability of any individual non-attainer falling in the same range with and being under the same scope as an attainer. Likewise, $Pr(E|B|X_\Phi)$ is the probability of any individual attainer falling in the same range with and being under the same scope as a non-attainer. These two probabilities need not always be equal. In fact, they are equal only when $\Phi = 0.50$.

What remains is to calculate the area of Section E (i.e., $\delta$). This is done in Appendix A.

## C. Results of the Analysis

Tables IV-3 and IV-4 give the probabilities of falling in Section E given attainment and of falling in Section E given non-attainment, respectively. These Tables are derived from the probability formulae in Table IV-2. To obtain the probabilistic marginal utilities of attainment, we simply perform a matrix subtraction, Table IV-4 minus Table IV-3. The results of this subtraction are shown in Table IV-5.

Note that the marginal utilities decrease for constant $\Phi$ and increasing $\alpha$, and decrease for constant $\alpha$ and increasing $\Phi$. Furthermore, each column, reflects about the row where $\Phi = 0.50$ so that each column below this row is the negative converse of the column above.

An inspection of Table IV-5 shows that it is not individually advantageous to obtain the high school diploma until 55% of the population under consideration (17-year old age cohort) does so. The row where $\Phi = 0.50$ can be considered to be the indifference level. However, a mean/median analysis shows that, in the aggregate, it is always advantageous to be an attainer rather than a non-attainer. This is so because for all values of $\alpha$, $\mu_{\Phi}$ is greater than $\mu_{\bar{\Phi}}$ (except when they are equal, when $\alpha = 0$). A complete mean/median analysis is given in Appendix B. See columns 4 and 6 in Table B-1.

This analysis of the Probabilistic Utility Model exposes an interesting paradox: in the aggregate it is more advantageous to be an attainer no matter what $\Phi$ and $\alpha$ are; individually this is not always the case. Furthermore, Table IV-5 indicates that the marginal disutility of not attaining the high school degree increases as attainment increases and also increases as the meritocratic parameter decreases! This phenomenon can be vividly seen in the lower left-hand quadrant of Table IV-5.

This quadrant corresponds to the decreasing power of the normative principle as the attainment rate increases toward 100%. As we move from the upper right-hand to the lower left-hand corner on the diagonal, disutilities can be seen to double, triple and even quadruple at various steps.

## TABLE IV-3

### PROBABILITY OF FALLING IN SECTION E GIVEN ATTAINMENT OF LEVEL 12

**Meritocratic Parameter (α)**

| Proportion of 12th Level Attainers(Φ) | 0.1000 | 0.2000 | 0.3000 | 0.4000 | 0.5000 | 0.6000 | 0.7000 | 0.8000 | 0.9000 | 0.9500 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0100 | 0.0035 | 0.0022 | 0.0015 | 0.0010 | 0.0007 | 0.0004 | 0.0002 | 0.0001 | 0.0000 | 0.0000 |
| 0.0500 | 0.0176 | 0.0112 | 0.0076 | 0.0051 | 0.0033 | 0.0021 | 0.0012 | 0.0005 | 0.0002 | 0.0001 |
| 0.1000 | 0.0353 | 0.0224 | 0.0152 | 0.0101 | 0.0067 | 0.0042 | 0.0023 | 0.0011 | 0.0003 | 0.0001 |
| 0.1500 | 0.0529 | 0.0336 | 0.0228 | 0.0152 | 0.0100 | 0.0062 | 0.0035 | 0.0016 | 0.0005 | 0.0002 |
| 0.2000 | 0.0706 | 0.0448 | 0.0304 | 0.0203 | 0.0134 | 0.0083 | 0.0047 | 0.0022 | 0.0006 | 0.0002 |
| 0.2500 | 0.0882 | 0.0560 | 0.0380 | 0.0254 | 0.0167 | 0.0104 | 0.0058 | 0.0027 | 0.0008 | 0.0003 |
| 0.3000 | 0.1058 | 0.0672 | 0.0456 | 0.0304 | 0.0200 | 0.0125 | 0.0070 | 0.0033 | 0.0010 | 0.0003 |
| 0.3500 | 0.1235 | 0.0785 | 0.0532 | 0.0355 | 0.0234 | 0.0146 | 0.0082 | 0.0038 | 0.0011 | 0.0004 |
| 0.4000 | 0.1411 | 0.0897 | 0.0608 | 0.0406 | 0.0267 | 0.0166 | 0.0094 | 0.0044 | 0.0013 | 0.0004 |
| 0.4500 | 0.1588 | 0.1009 | 0.0684 | 0.0456 | 0.0301 | 0.0187 | 0.0105 | 0.0049 | 0.0014 | 0.0005 |
| 0.5000 | 0.1764 | 0.1121 | 0.0759 | 0.0507 | 0.0334 | 0.0208 | 0.0117 | 0.0055 | 0.0016 | 0.0005 |
| 0.5500 | 0.1940 | 0.1233 | 0.0835 | 0.0558 | 0.0367 | 0.0229 | 0.0129 | 0.0060 | 0.0018 | 0.0006 |
| 0.6000 | 0.2117 | 0.1345 | 0.0911 | 0.0608 | 0.0401 | 0.0250 | 0.0140 | 0.0066 | 0.0019 | 0.0006 |
| 0.6500 | 0.2293 | 0.1457 | 0.0987 | 0.0659 | 0.0434 | 0.0270 | 0.0152 | 0.0071 | 0.0021 | 0.0007 |
| 0.7000 | 0.2470 | 0.1569 | 0.1063 | 0.0710 | 0.0468 | 0.0291 | 0.0164 | 0.0077 | 0.0023 | 0.0007 |
| 0.7500 | 0.2646 | 0.1681 | 0.1139 | 0.0760 | 0.0501 | 0.0312 | 0.0176 | 0.0082 | 0.0024 | 0.0008 |
| 0.8000 | 0.2822 | 0.1793 | 0.1215 | 0.0811 | 0.0534 | 0.0333 | 0.0187 | 0.0088 | 0.0026 | 0.0008 |
| 0.8500 | 0.2999 | 0.1905 | 0.1291 | 0.0862 | 0.0568 | 0.0354 | 0.0199 | 0.0093 | 0.0027 | 0.0009 |
| 0.9000 | 0.3175 | 0.2017 | 0.1367 | 0.0913 | 0.0601 | 0.0374 | 0.0211 | 0.0099 | 0.0029 | 0.0009 |
| 0.9500 | 0.3352 | 0.2130 | 0.1443 | 0.0963 | 0.0635 | 0.0395 | 0.0222 | 0.0104 | 0.0031 | 0.0010 |

## TABLE IV-4

### PROBABILITY OF FALLING IN SECTION E GIVEN ATTAINMENT BELOW LEVEL 12

**Meritocratic Parameter (α)**

| Proportion of 12th Level Attainers(Φ) | 0.1000 | 0.2000 | 0.3000 | 0.4000 | 0.5000 | 0.6000 | 0.7000 | 0.8000 | 0.9000 | 0.9500 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0100 | 0.3493 | 0.2219 | 0.1504 | 0.1004 | 0.0661 | 0.0412 | 0.0232 | 0.0109 | 0.0032 | 0.0010 |
| 0.0500 | 0.3352 | 0.2130 | 0.1443 | 0.0963 | 0.0635 | 0.0395 | 0.0222 | 0.0104 | 0.0031 | 0.0010 |
| 0.1000 | 0.3175 | 0.2017 | 0.1367 | 0.0913 | 0.0601 | 0.0374 | 0.0211 | 0.0099 | 0.0029 | 0.0009 |
| 0.1500 | 0.2999 | 0.1905 | 0.1291 | 0.0862 | 0.0568 | 0.0354 | 0.0199 | 0.0093 | 0.0027 | 0.0009 |
| 0.2000 | 0.2822 | 0.1793 | 0.1215 | 0.0811 | 0.0534 | 0.0333 | 0.0187 | 0.0088 | 0.0026 | 0.0008 |
| 0.2500 | 0.2646 | 0.1681 | 0.1139 | 0.0760 | 0.0501 | 0.0312 | 0.0176 | 0.0082 | 0.0024 | 0.0008 |
| 0.3000 | 0.2470 | 0.1569 | 0.1063 | 0.0710 | 0.0468 | 0.0291 | 0.0164 | 0.0077 | 0.0023 | 0.0007 |
| 0.3500 | 0.2293 | 0.1457 | 0.0987 | 0.0659 | 0.0434 | 0.0270 | 0.0152 | 0.0071 | 0.0021 | 0.0007 |
| 0.4000 | 0.2117 | 0.1345 | 0.0911 | 0.0608 | 0.0401 | 0.0250 | 0.0140 | 0.0066 | 0.0019 | 0.0006 |
| 0.4500 | 0.1940 | 0.1233 | 0.0835 | 0.0558 | 0.0367 | 0.0229 | 0.0129 | 0.0060 | 0.0018 | 0.0006 |
| 0.5000 | 0.1764 | 0.1121 | 0.0759 | 0.0507 | 0.0334 | 0.0208 | 0.0117 | 0.0055 | 0.0016 | 0.0005 |
| 0.5500 | 0.1588 | 0.1009 | 0.0684 | 0.0456 | 0.0301 | 0.0187 | 0.0105 | 0.0049 | 0.0014 | 0.0005 |
| 0.6000 | 0.1411 | 0.0897 | 0.0608 | 0.0406 | 0.0267 | 0.0166 | 0.0094 | 0.0044 | 0.0013 | 0.0004 |
| 0.6500 | 0.1235 | 0.0785 | 0.0532 | 0.0355 | 0.0234 | 0.0146 | 0.0082 | 0.0038 | 0.0011 | 0.0004 |
| 0.7000 | 0.1058 | 0.0672 | 0.0456 | 0.0304 | 0.0200 | 0.0125 | 0.0070 | 0.0033 | 0.0010 | 0.0003 |
| 0.7500 | 0.0882 | 0.0560 | 0.0380 | 0.0254 | 0.0167 | 0.0104 | 0.0058 | 0.0027 | 0.0008 | 0.0003 |
| 0.8000 | 0.0706 | 0.0448 | 0.0304 | 0.0203 | 0.0134 | 0.0003 | 0.0047 | 0.0022 | 0.0006 | 0.0002 |
| 0.8500 | 0.0529 | 0.0336 | 0.0228 | 0.0152 | 0.0100 | 0.0062 | 0.0035 | 0.0016 | 0.0005 | 0.0002 |
| 0.9000 | 0.0353 | 0.0224 | 0.0152 | 0.0101 | 0.0067 | 0.0042 | 0.0023 | 0.0011 | 0.0003 | 0.0001 |
| 0.9500 | 0.0176 | 0.0112 | 0.0076 | 0.0051 | 0.0033 | 0.0021 | 0.0012 | 0.0050 | 0.0002 | 0.0001 |



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## TABLE IV-5

### PROBABILISTIC MARGINAL UTILITIES OF ATTAINMENT OF LEVEL 12

**Meritocratic Parameter (α)**

| Proportion of 12th Level Attainers(Φ) | 0.1000 | 0.2000 | 0.3000 | 0.4000 | 0.5000 | 0.6000 | 0.7000 | 0.8000 | 0.9000 | 0.9500 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0100 | 0.3460 | 0.2200 | 0.1490 | 0.0990 | 0.0650 | 0.0410 | 0.0230 | 0.0110 | 0.0030 | 0.0010 |
| 0.0500 | 0.3180 | 0.2020 | 0.1370 | 0.0910 | 0.0600 | 0.0370 | 0.0210 | 0.0100 | 0.0030 | 0.0010 |
| 0.1000 | 0.2820 | 0.1790 | 0.1220 | 0.0810 | 0.0530 | 0.0330 | 0.0190 | 0.0090 | 0.0030 | 0.0010 |
| 0.1500 | 0.2470 | 0.1570 | 0.1060 | 0.0710 | 0.0470 | 0.0290 | 0.0160 | 0.0080 | 0.0020 | 0.0010 |
| 0.2000 | 0.2120 | 0.1340 | 0.0910 | 0.0610 | 0.0400 | 0.0250 | 0.0140 | 0.0070 | 0.0020 | 0.0010 |
| 0.2500 | 0.1760 | 0.1120 | 0.0760 | 0.0510 | 0.0330 | 0.0210 | 0.0120 | 0.0050 | 0.0020 | 0.0010 |
| 0.3000 | 0.1410 | 0.0900 | 0.0610 | 0.0410 | 0.0270 | 0.0170 | 0.0090 | 0.0040 | 0.0010 | 0.0000 |
| 0.3500 | 0.1060 | 0.0670 | 0.0460 | 0.0300 | 0.0200 | 0.0120 | 0.0070 | 0.0030 | 0.0010 | 0.0000 |
| 0.4000 | 0.0710 | 0.0450 | 0.0300 | 0.0200 | 0.0130 | 0.0080 | 0.0050 | 0.0020 | 0.0010 | 0.0000 |
| 0.4500 | 0.0350 | 0.0220 | 0.0150 | 0.0100 | 0.0070 | 0.0040 | 0.0020 | 0.0010 | 0.0000 | 0.0000 |
| 0.5000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.5500 | -0.0350 | -0.0220 | -0.0150 | -0.0100 | -0.0070 | -0.0040 | -0.0020 | -0.0010 | 0.0000 | 0.0000 |
| 0.6000 | -0.0710 | -0.0450 | -0.0300 | -0.0200 | -0.0130 | -0.0080 | -0.0050 | -0.0020 | -0.0010 | 0.0000 |
| 0.6500 | -0.1060 | -0.0670 | -0.0460 | -0.0300 | -0.0200 | -0.0120 | -0.0070 | -0.0030 | -0.0010 | 0.0000 |
| 0.7000 | -0.1410 | -0.0900 | -0.0610 | -0.0410 | -0.0270 | -0.0170 | -0.0090 | -0.0040 | -0.0010 | 0.0000 |
| 0.7500 | -0.1760 | -0.1120 | -0.0760 | -0.0510 | -0.0330 | -0.0210 | -0.0120 | -0.0050 | -0.0020 | -0.0010 |
| 0.8000 | -0.2120 | -0.1340 | -0.0910 | -0.0610 | -0.0400 | -0.0250 | -0.0140 | -0.0070 | -0.0020 | -0.0010 |
| 0.8500 | -0.2470 | -0.1570 | -0.1060 | -0.0710 | -0.0470 | -0.0290 | -0.0160 | -0.0080 | -0.0020 | -0.0010 |
| 0.9000 | -0.2820 | -0.1790 | -0.1220 | -0.0810 | -0.0530 | -0.0330 | -0.0190 | -0.0090 | -0.0030 | -0.0010 |
| 0.9500 | -0.3180 | -0.2020 | -0.1370 | -0.0910 | -0.0600 | -0.0370 | -0.0210 | -0.0100 | -0.0030 | -0.0010 |

## V. RESULTS, CONCLUSIONS, CONJECTURES and POLICY ALTERNATIVES

These models illustrate the theoretical limitations of education policy designed to increase the high school attainment rate to 90% or above and to help minorities share in the "benefits" of educational attainment. They are formal models and are not grounded in empirical results. Like Raymond Boudon's models (see Part VI), they avoid the cross-sectional and variable confounding of survey data. They illustrate the power of a logical tautology in conjunction with a normative principle. However, these idealized models are not without limitations

The Aggregate Model seems, on the face of it, too meritocratic for our present society. The distribution of social benefits may not in reality, be normal and their means (as shown in the Individual Utility Model) may not remain constant with systemic growth (which is clearly not the case in the Aggregate Model). Nonetheless, these models can serve as "benchmarks" against which to compare other logico-mathematical models containing different assumptions, and still others based upon empirically derived data. They also add to our database of models.

***Policy Alternatives.*** The results of the models developed in this analysis suggest possible alternative education policy scenarios.

***Push the High School Attainment Rate to 100% quickly.*** Given that attempts to reduce social inequalities by increasing the national high school attainment ratio will fail, what would be the consequences of entirely eliminating educational attainment inequality at the high school level? That is, push the high school attainment rate to 100% so that the high school diploma can no longer be the basis for the distribution of non-educational social goods.

This approach has two major pitfalls. First, the system had better reach 100% attainment very quickly so as to minimize the hardships that will have to be endured by the ever decreasing percentage of non-attainers. Second, even if such a result could be achieved, the original inequality problems would remain unsolved since the problems would merely be shifted to the next higher level of the educational system: postsecondary.

If the normative principle persists (and there is no reason to assume that it will not) then the distributional instrument of social goods will shift to the postsecondary level. This level is, for the most part, selective. One does not only choose to go on, one is chosen. Thus, enormous pressures will come to bear upon this level to alter its selectivity feature. One can argue that this pressure is already fairly strong.

***Reduce the High School Attainment Rate to the 55-60% Level.*** This level is below the "equilibrium point" of the Aggregate Model and close to the "indifference" level of the Utility Model. This is the point at which the effects of the decline in the social benefits of attainment and the precipitous rise in the social liabilities of non-attainment are (theoretically) thought to begin.

Of course, careful consideration needs to be given to the provision of ample opportunities for all to continue their education (i.e., pursue learning). Such a policy must avoid an inequitable distribution of the non-attainers based on educationally irrelevant attributes such as race, class and ethnic background. Admittedly, a policy of this sort would not enjoy widespread political support.

***Abandon the Normative Principle.*** The two previous alternatives assumed the continued presence of the normative principle. But what would life be like without it? The abandonment of this principle might be the most efficacious, but a politically and socially difficult, way to reduce educational and socioeconomic inequality.

If educational attainment is no longer used as an instrument for the distribution of non-educational social goods, then perhaps education could once again be pursued for the benefits that are intrinsic in the educational goods themselves and not for the socioeconomic advantages that disappear and reappear with ever increasing rates and different levels of attainment.

Such a move might signal the end of the illusion that the educational system is a solution to practically every social ill. I do not claim to know just what new instruments for the distribution of social benefits would arise, nor how one could go about judging their desirability as a replacement for educational attainment. However, a reconsideration of the socioeconomic normative principle that disproportionately rewards formal educational attainment might prove to be a beneficial exercise.

## VI. ANALYTICAL POSTSCRIPT: BOUDON'S MODELS OF INEQUALITY OF EDUCATIONAL and SOCIAL OPPORTUNITY

Two models created by the French Sociologist, Raymond Boudon (Boudon, 1974) support the results of the two models presented in this paper. Boudon's models are of inequality of educational opportunity (IEO) and inequality of social opportunity (ISO). He analyses their relationship to one another and to the educational and social systems. Some of Boudon's relevant results and analyses follow.

### A. Boudon's IEO and ISO Models and the Theory of Educational Systems

Boudon's models and his analyses are highly suggestive in many ways. In addition to a methodological approach which avoids some of the pitfalls of factorial analysis (i.e., partial accounting for total variance, cross-sectional "illusions," and lack of quantitatively adequate data), Boudon adds an important dimension to the description of the normative behavior of the type of educational system spawned in Western industrial societies. This dimension, system animation, is of fundamental import in helping to provide a clear and precise picture of the dynamics of systemic motion.

By observing (and modeling) the over-time cumulative effects of the various factors affecting the educational system's growth, Boudon is able to discern the logical limits and consequences of this growth. The ceiling-effect and the exponential mechanism that combine to drive the IEO model help generate a number of observations and paradoxes that bear significantly upon the theory of educational systems as presented in this paper.

#### *Some Familiar Paradoxes*

One of the most striking paradoxes generated by Boudon's models is that "other things being equal" (which is seldom the case), educational growth has the effect of increasing social and economic inequality. This happens even when the system becomes more egalitarian with respect to educational opportunity (EO).

This paradox rests upon the assumption that income is dependent upon educational attainment level. Over time, educational level and socioeconomic status increase with educational level increasing more rapidly the higher the socioeconomic level. Since both of these factors are "independently" responsible for income differentials, "economic inequality will increase over time along with social inequality, for the latter is correlated with the former." (Boudon, 1974, pg. 188)

The paradox is completed when we add another important conclusion reached by the application of Boudon's model: change in social stratification is the only factor that can substantially affect the model's exponential mechanism and hence ISO. This leads Boudon to conclude that educational growth can partially explain the "persistence of economic inequality in Western societies." (ibid., 188) It is quite remarkable that Boudon's model and the models pesented in this paper reach identical conclusions using such different but complementary methods.

***The Success-Breeds-Futility Paradox***

Another paradox illustrates just how the apparent success of the educational system leads to futility for some participants and how the system fuels the fires of its own expansion.

Boudon's models indicate that one of the main endogenous factors responsible for the increase in educational demand is the over-time change in the status expectations of individuals with respect to educational level.

> ...as time goes on, the structure of expectations associated with the two highest levels of education is constant; intermediate levels are affected most adversely; the structure of expectations relating to the lowest levels of education becomes less favorable, too, but it is less influenced by the overall educational increase than are the intermediary levels. (ibid., 149)

Thus, as IEO decreases over time and the educational system expands at all levels, the social status expectations for persons at intermediate educational levels decrease and these persons must raise their levels just to maintain constant social status expectations. This treadmill effect means that while the relation between educational level and social status changes very little over time, the number of years of schooling associated with each of the educational levels increases.

Thus, while the average level of educational attainment in the population increases, the educational levels that are associated with particular status expectations are "simultaneously moving upward." As individuals demand more and more education over time, the individual return tends to be nil, while the aggregate return on this demand is high. The lower socioeconomic classes are compelled to demand more education (especially if the higher classes to do), for not to do so condemns these lower classes to constantly falling social status expectations. However, more educational demand only retards this diminution in status and does not increase the lower class's chances of achieving increased social status.

This is a particularly frustrating paradox, for in a meritocratic society where the normative principle holds, an individual seems to have an advantage in securing as much education as he or she can. However, when many individuals seek additional education, the aggregate effects of this demand decrease the social status expectations associated with most of the educational levels This causes people to demand even more education in the next time period.

This paradox lends support to a number of results due to the interactions between various systemic principles such as the Law of Zero Correlation, the Principle of Shifting Benefits and Liabilities, the Law of Last Entry, and the Principle of the Moving Target. Boudon shows that when expectations associated with some particular educational level become reduced, a decrease in expectations at all levels results. (ibid., Table 8.4, 147)

Boudon sees evidence that this point has been reached at the secondary level in some industrial societies, but "it seems that not even the most advanced industrial societies have achieved a proportion of college students so large that a severe decrease in the expectations at this level can

be observed." (ibid., 150) One wonders whether or not the American educational system has moved to a point beyond Boudon's claim? Because of their logico-mathematical nature, the models presented in this paper are generalizable over all systemic levels. Already, over 60% of the high school graduates enter higher educational institutions (National Center for Education Statistics, 1994). It may not be long before the system approachs zero correlation at this level!

Perhaps in anticipation of zero correlation at the college level, Thurow has called for a "system of post-secondary education for the non-college bound student" (Thurow, 1994). However, I suggest that such a "system" (even if established independently of the educational system) would itself be absorbed into the educational system and therefore be subject to its laws and thus perpetuate the paradoxes discussed in this paper. Such is the power of the dynamics of the educational system.[10]

**B. Further Observations on Systemic Growth**

While the paradoxes generated by Boudon's model are important for establishing the boundaries and limitations of educational systems, there are other observations on growth that warrant exploration.

Boudon, in his Appendix to Chapter 9, indicates that by manipulating the demand for education (i.e., predicating demand in the educational system upon exogenous rather than endogenous factors), equality of educational opportunity (EEO) can be affected. This is the only alternative, other than changing social stratification, that he offers to remedy IEO and ISO.

Now, if the number of positions (student slots) in the educational system at the highest level remains unchanged and if the number of positions at the middle level is increased by D during time period t to t+1, and if the number of positions at the lowest level is decreased by D during this same time period --then, how is the number of persons with lowest social background ($T_t$) who reach at least the middle educational level affected by the value of D?

Boudon concludes on the basis of this "modified" model that $T_t$ is an increasing function of time and an increasing function of D. Furthermore, $T_t$ increases at a decreasing rate as a function of process-phase. According to Boudon, the duration's of the three phases are a function of D ("an increase in D has the effect of shortening the first and second phases..."). Thus non-linear returns in $T_t$ are associated with increase in the value of D. This thesis is presented in expanded form in (Boudon, 1976).

This "modified" model (reflecting an "ideal-typical planned educational system") results in a decrease in IEO through the manipulation of demand, while the IEO parameter, "a", remains constant over time. (This IEO parameter has marked similarities to the meritocratic parameter, $\alpha$, presented in the Aggregate and Individual Utility models.) The free-market endogenous educational system creates what appear to be insurmountable problems (i.e., the paradoxes)

On the other hand, the exogenous educational system, permits us in theory at least, to correct some of these undesirable effects. Boudon rightfully questions the high social costs of this remedy. Nevertheless, this "modified" model may provide additional insights into the growth mechanism of the system and may have enormous implications for policy and planning especially if the demand for education is to be controlled. It deserves further study.

### C. A Logistic Growth Curve

In an intriguing footnote (ibid., 201, ff.3), Boudon suggests that in conjunction with the paradoxes cited above, there is a particular point in the free-market educational system development where "growth is more rapid at the higher level than at the secondary level and thus a decrease in IEO and ISO is curtained." (ibid., 199) This growth, fueled by unrestrained demand or more education, may lead to a state of "latent crisis." This runaway exponential growth trend may be checked by a "braking process" that is proportional to the trend, leading to a logistic rather than an exponential growth curve.

What are the circumstances that would lead to this braking process and would these circumstances be endogenous or exogenous to the educational system? The answers to these questions are fundamental to education policy. These answers appear to be intimately related to many of the systemic principles in the general theory of educational systems.

Finally, what is to be made of Boudon's enigmatic statement that "the concern of all industrial societies with short-term higher education can be better understood in the light of the dialectic between the exponential growth of educational demand and the (proportional) braking process...?" Perhaps the theory of educational systems and the models presented in this paper can shed some light on this question?

## NOTES

**1.** Some of the ideas and models presented in this paper have appeared in various forms and stages of completion in previous works of mine. In particular: Thomas F. Green(1980) with David P. Ericson and Robert H. Seidman; Seidman(1982); Seidman(1981). The analysis of Boudon's work has never been published.

**2.** This paper uses the high school attainment rate as the measure of systemic "size due to growth in attainment." One reason is that this is what the Congressional Act focuses on. Another, is that the 12th grade is the last level of the educational system that is non-selective. For the most part, one not only chooses to go on to post-secondary education, one is chosen. It is this fact, together with certain systemic laws, that illustrates the inherent futility of certain education policies at particular stages of systemic growth.

I use the 17 year-old age-cohort to measure the high school attainment rate. This is the cohort used by the National Center for Educationl Statistics (1994) to track the high school attainment ratio since 1869. The models presented in this paper are based upon a dichotomized population: those who have not completed high school and those who have but have not gone on to the post-secondary level of the system.

However, some researchers use a different age-cohort. For example, the National Education Goals Panel uses the 19-20 year-old age cohort (National Education Goals Panel, September 1994). Other studies report high school completion rates amongst various age cohorts, including 21-22 year-olds and even 29-30 year-olds (McMillen, 1993).The numeric ratios will differ, of course. A standard measure of high school "completion and school leaving" has been proposed. The "appropriate unit of analysis" is the graduating class cohort.(Hartzell, 1992).

**3.** A sample calculation can be found in Appendix C.

**4.** It is probably unreasonable to apply the model at the lower attainment rates where the power of the normative principle is very low. However, the model does serve to illustrate the idea that the relative benefit disparity between the two groups first decreases and then increases. This phenomenon suggests that a particular educational policy appropriate for one stage of systemic growth may not be appropriate for another.

**5.** U.S. Bureau of the Census, Decenial Census Reports for 1940, 1950, 1960, 1970; Current Population Reports, P60, nos. 85, 90, 92, 97, 101. U. S. Bureau of the Census, Current Population Reports for 1984, 1987, 1990; P70, nos. 11, 21, 32 ("Educational Background and Economic Status").

**6.** See Levin(1972) for a traditional analysis of the relevant data.

**7.** For an extended analysis from another methodological perspective, see Appendix C in (Green, 1980).

**8.** See the Table reproduced in Appendix D (National Center for Education Statistics, 1994). It is interesting to note that the U.S. Government projection of the high school attainment ratio to the year 2004 keeps it steady at about 77% (using the 18 year-old cohort). Why? No reason is given. See Tables 26 and B4 (National Center for Education Statistics, 1993).

**9.** This irony (in the form of paradoxes) is addressed by Boudon (1974) and is analyzed in Part VI in this paper. Boudon's models confirm the results of the Aggregate and Individual models.

**10.** For an example of such an absorption scenario, see Seidman's (1982) analysis of the "lifelong learning system."

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## APPENDIX A

## CALCULATIONS OF SECTION E AREA

To calculate δ, we begin by truncating the asymptotes of the two standardized normal curves (Figure IV-1) at 3.0 standard deviations above and below their respective means. As a result, we lose 0.26% of the population of any one curve.

Since the two curves are identical (i.e., both are standardized normal curves), the point on the X-axis ($\mu_1$ directly below the point of intersection, I) lies midway between the $X_\Phi$ and $X_\Phi$ distribution means, $\mu_\Phi$ and $\mu_\Phi$, respectively. This follows from the laws of symmetry, since Section D is always equal to Section A in area. Figure A-1 emphasizes the area of intersection in Figure IV-1.

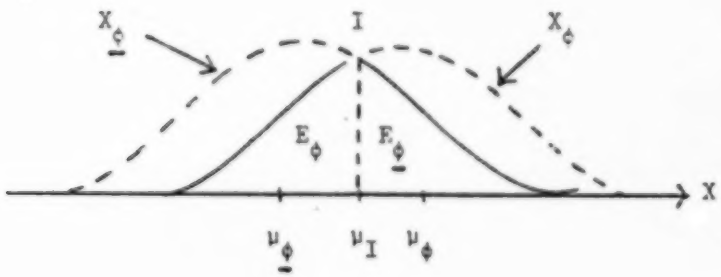


**Figure A-1 Section E Area Emphasized**
($E_\Phi$ and $E_\Phi$ correspond to $E_1$ and $E_2$, respectively, in Table IV-2)

We know by symmetry, that the area to the right of the vertical line $I\mu_I$ on curve $X_\phi$ (i.e., area $E_\phi$) is equal to the area to the left of $I\mu_I$ on curve $X_\phi$ (i.e., area $E_\phi$). Thus, twice $E_\phi$ or twice $E_\phi$ gives us $\delta$, the area of Section $E_\phi$.

Now we can proceed to develop a pair of algorithms that enable us to calculate area $E_\phi$.

The area $\delta$, equals 1.0 when $\alpha$ equals zero. In this situation, $X_\phi$ and $X_\phi$ are superimposed one upon the other. Since $\mu_\phi = \mu_\phi$, their relative difference, $\Psi$, is equal to $|\mu_\phi - \mu_\phi|$ which is equal to zero. When $\alpha = 1.0$, area $\delta$ equals zero. In this case, $X_\phi$ and $X_\phi$ are mutually exclusive and $\Psi$ equals 6.0. Between these two extremes, $\alpha$ ranges from zero to 1.0.

We first examine the case where $\alpha$ ranges from zero to 0.5 and then the case where $\alpha$ ranges from 0.5 to 1.0. (Note that 0.5 is used throughout as an approximation to 0.4987, which is used in the calculations due to truncation.)

**CASE 1: ($0 <= \alpha => 0.5$)**

Consider Figure A-2. The relative distance, $\Psi$, between the two means, $\mu_\phi$ and $\mu_\phi$, is equal to the distance on the X-axis under area A (i.e., the area corresponding to the value of $\alpha$).

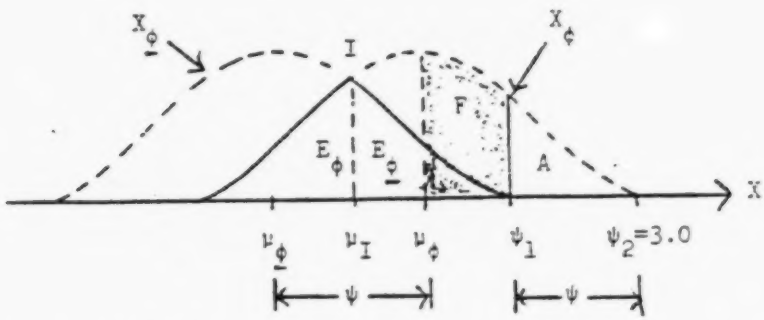


**Figure A-2. Case 1: Where $\alpha$ Ranges from 0 to 0.5**

Note that when $\alpha = 0$, the two means, $\mu_\phi$ and $\mu_\phi$, coincide simply because the two curves, $X_\phi$ and $X_\phi$, are superimposed one upon the other. As the value of $\alpha$ increases, the $X_\phi$ curve is shifted to the left, a distance equal to the distance on the X-axis under Section A. Call this distance $\Psi$, which is the value of the $X_\phi$ curve translation.

Since $\Psi_2 = 3.0$, we need only find $\Psi_1$ in order to find $\Psi$ (i.e., $\Psi = \Psi_2 - \Psi_1$). Area F is equal to 0.4987 - $\alpha$ and $\Psi_1$ is found from a standardized normal curve table. Once we have computed $\Psi$, we can locate $\mu_1$ with respect to $\mu_\Phi$. See Figure A-3.

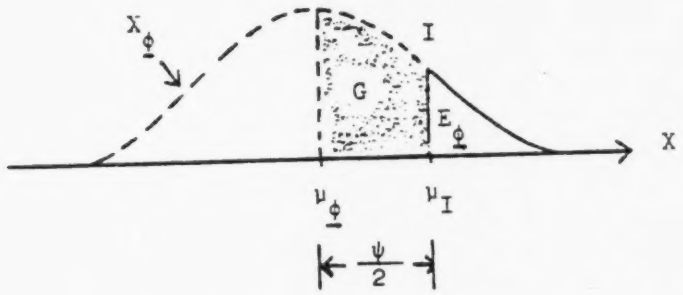


**Figure A-3. The Parameters for Finding δ**

Note that $\mu_1$ lies $\Psi/2$ above $\mu_\Phi$. Area G is found from a standardized normal curve table. Area $E_\Phi$ is equal to 0.4987 - G. The area $\delta$, is simply twice area $E_\Phi$. The algorithm for this computation is shown in Algorithm A-1.

## ALGORITHM A-1

### CASE 1: WHERE α RANGES FROM 0 TO 0.5
**(Refer to Figures A-2 and A-3)**

| Step | |
|---|---|
| 1 | $F = 0.4987 - \alpha$ |
| 2 | $\Psi_1$ from standardized normal curve table |
| 3 | $\Psi = \Psi_2 - \Psi_1$ |
| 4 | $\mu_1 = \Psi/2$ with respect to $\mu_\Phi$ |
| 5 | G from standardized normal curve table |
| 6 | $E_\Phi = 0.4987 - G$ |
| 7 | $\delta = 2(E_\Phi)$ |

### CASE 2: (0.5 <= α => 1.0)

Figure A-4 depicts the situation for this case, and the algorithm for the computation of δ is shown in Algorithm A-2.

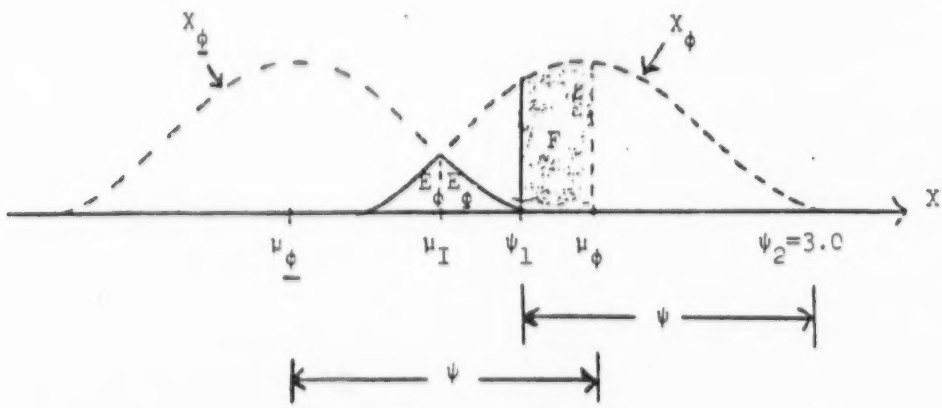


**Figure A-4. Case 2: Where α Ranges from 0.5 to 1.0**

## ALGORITHM A-2

### CASE 2: WHERE $\alpha$ RANGES FROM 0.5 TO 1.0
**(Refer to Figures A-3 & A-4)**

| Step | |
|---|---|
| 1 | $F = \alpha - 0.4987$ |
| 2 | $\Psi_1$ from standardized normal curve table |
| 3 | $\Psi = \Psi_2 + \Psi_1$ |
| 4 | $\mu_1 = \Psi/2$ with respect to $\mu_\Phi$ |
| 5 | G from standardized normal curve table |
| 6 | $E_\Phi = 0.4987 - G$ |
| 7 | $\delta = 2(E_\Phi)$ |

Table A-1, gives the values of $\delta$ for $\alpha$ values in steps of 0.1. Table A-2 gives the intermediate values of $F, \Psi_1, \Psi, \mu_1, G, \mu_\Phi$ for $\alpha$ values in steps of 0.1.

**TABLE A-1**

**VALUES OF δ AS A FUNCTION OF α**

| α | δ |
|---|---|
| 0 | 1.0000 |
| 0.10 | 0.3872 |
| 0.20 | 0.2776 |
| 0.30 | 0.2124 |
| 0.40 | 0.1666 |
| 0.50 | 0.1310 |
| 0.60 | 0.1006 |
| 0.70 | 0.0750 |
| 0.80 | 0.0516 |
| 0.90 | 0.0294 |
| 0.95 | 0.0172 |
| 1.00 | 0 |

**TABLE A-2**

**INTERMEDIATE VALUES FROM ALGORITHMS A-1 and A-2**

| α | F | $\Psi_1$ | Ψ | $\mu_1$ | G | $E_e$ |
|---|---|---|---|---|---|---|
| 0 | | | 0 | | | |
| 0.10 | 0.3987 | 1.275 | 1.725 | 0.8625 | 0.3051 | 0.1936 |
| 0.20 | 0.2987 | 0.835 | 2.165 | 1.0825 | 0.3599 | 0.1388 |
| 0.30 | 0.1987 | 0.520 | 2.480 | 1.2400 | 0.3925 | 0.1062 |
| 0.40 | 0.0987 | 0.250 | 2.750 | 1.3750 | 0.4154 | 0.0833 |
| 0.50 | 0 | 0 | 3.000 | 1.5000 | 0.4332 | 0.0655 |
| 0.60 | 0.1013 | 0.255 | 3.255 | 1.6275 | 0.4484 | 0.0503 |
| 0.70 | 0.2013 | 0.530 | 3.530 | 1.7650 | 0.4612 | 0.0375 |
| 0.80 | 0.3013 | 0.850 | 3.850 | 1.9250 | 0.4729 | 0.0258 |
| 0.90 | 0.4013 | 1.290 | 4.290 | 2.1450 | 0.4840 | 0.0147 |
| 0.95 | 0.4513 | 1.660 | 4.660 | 2.3300 | 0.4901 | 0.0086 |
| 1 | | | 6.000 | | | |

## APPENDIX B

## MEAN/MEDIAN ANALYSIS OF THE PROBABILISTIC UTILITY MODEL

We can set the Model in motion. See Figure B-1. Note that when $\alpha = 0$, the following equalities hold:

$$\mu_B = \mu_C = \mu_I = \mu_{\Phi} = \mu_{\Phi} \quad (1)$$

$$| \mu_A - \mu_I | = | \mu_D - \mu_I | \quad (2)$$

When $\alpha = 1$, another set of equalities hold:

$$\mu_C = \mu_B = \mu_I \quad (3)$$

$$\mu_A = \mu_{\Phi} \quad (4)$$

$$\mu_D = \mu_{\Phi} \quad (5)$$

$$| \mu_A - \mu_I | = | \mu_D - \mu_I | \quad (6)$$

Between these two extremes, it is possible to calculate the relative differences between medians ($\mu_{\Phi}$ and $\mu_{\Phi}$ are the grand means and grand medians of their respective distributions) of the various sections of the two curves shown in Figure B-1.

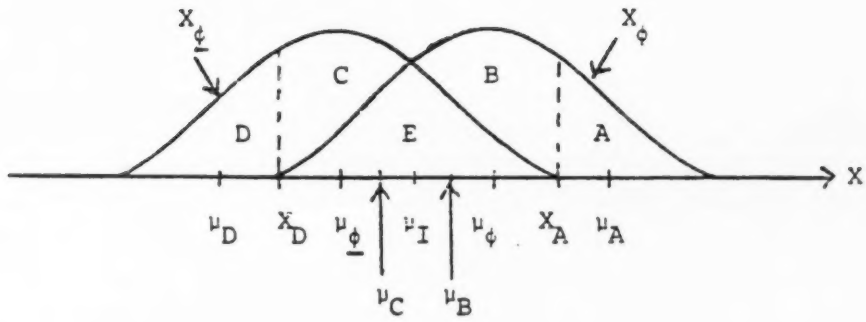


**Figure B-1. Medians/Means for Sections of Curves**

Assume that $\mu_\Phi$ remains constant and that both curves retain their normal shapes as the size of $\Phi$ (and concomitantly, $\underline{\Phi}$) and $\alpha$ change. We take $\mu_\Phi$ as our point of reference, since it remains constant, and calculate the other medians with respect to it.

## 1. Schema's for Median Calculations for Changing Values of $\alpha$

We begin, as we did in Appendix A, by truncating the asymptotes of the two standardized normal curves at 3.0 standard deviations above and below their respective means. Medians $\mu_A$ and $\mu_B$ have already been calculated in the Aggregate Model and can be found in columns 2 and 3 of Table III-1.

$\mu_\Phi$ is the distance on the X-axis under Section A. This distance is the $\Psi$ value computed as an intermediate step by Algorithms 1 and 2. See Table A-2. $\mu_I$ is simply one half $\mu_\Phi$ and is also computed as an intermediate step by Algorithms 1 and 2. See Table A-2.

We now develop schemas that compute the values of $\mu_C$ and $\mu_D$, for changing values of $\alpha$.

Due to the symmetry of the two curves and the equality of Sections A and D, median $\mu_C$ will always be as much to the right of $\mu_{\underline{\Phi}}$ as $\mu_B$ is to the left of $\mu_\Phi$. Thus,

$$\mu_C = \mu_{\underline{\Phi}} - \mu_B \qquad (7)$$

In a similar fashion, $\mu_D$ will always be as much to the left of $\mu_{\underline{\Phi}}$ as $\mu_A$ is to the right of $\mu_\Phi$. Thus,

$$\mu_D = \mu_{\underline{\Phi}} - \mu_A \qquad (8)$$

Table B-1 displays the results of these computations.

## 2. Changing Means ($\mu_{\Phi}$ and $\mu_{\bar{\Phi}}$) With Changing $\Phi$ and Constant $\alpha$

We have assumed throughout that the size of $\Phi$ has no effect upon the means of the dichotomized populations. Furthermore, for computational purposes, we have assumed that only $\mu_{\bar{\Phi}}$ was affected by changing $\alpha$ and that $\mu_{\Phi}$ remains permanently anchored.

It is not unreasonable to assume that both means change with changing $\Phi$ and that both means change with changing $\alpha$. However, both of these cases reduce to the analysis that has already been performed for the probability distributions generated by the formulae in Table IV-2 (constant $\mu_{\phi}$ for changing $\Phi$ and changing $\alpha$).

## TABLE B-1

## INTERMEDIATE VALUES FROM ALGORITHMS 1 AND 2

| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|
| $\alpha$ | $\mu_\Phi$ | $\mu_A$ | $\mu_B$ | $\mu_I$ | $\mu_C$ | $\mu_\Phi$ | $\mu_D$ |
| 0 | 0 | 3.0 | 0 | 0 | 0 | 0 | -3.0 |
| 0.10 | 0 | 1.645 | -0.126 | -0.8625 | -1.5990 | -1.725 | -3.370 |
| 0.20 | 0 | 1.283 | -0.253 | -1.0825 | -1.9120 | -2.165 | -3.448 |
| 0.30 | 0 | 1.037 | -0.385 | -1.2400 | -2.0950 | -2.480 | -3.517 |
| 0.40 | 0 | 0.842 | -0.524 | -1.3750 | -2.2260 | -2.750 | -3.592 |
| 0.50 | 0 | 0.675 | -0.675 | -1.5000 | -2.3250 | -3.000 | -3.675 |
| 0.60 | 0 | 0.524 | -0.842 | -1.6275 | -2.4130 | -3.255 | -3.779 |
| 0.70 | 0 | 0.385 | -1.037 | -1.7650 | -2.4930 | -3.530 | -3.915 |
| 0.80 | 0 | 0.253 | -1.283 | -1.9250 | -2.5670 | -3.850 | -4.103 |
| 0.90 | 0 | 0.126 | -1.645 | -2.1450 | -2.6450 | -4.290 | -4.416 |
| 0.95 | 0 | 0.063 | -1.960 | -2.3300 | -2.7000 | -4.660 | -4.723 |
| 1.0 | 0 | 0 | -3.0 | -3.0 | -3.0 | -6.0 | -6.0 |

To construct the probability tables for changing means, we can use the probability distributions generated by the formulae in Table IV-2. We need only know the sizes of $\Phi$ and $\alpha$, and the relative difference between the two dichotomized population means (see Appendix A). This relative difference, $| \mu_{\Phi} - \mu_{\bar{\Phi}} |$, is a function only of the size of $\alpha$. Thus, if both means change with changing $\Phi$ and with changing $\alpha$, and if we know the relative difference between the means, we can calculate the new $\alpha$. We can then consult the existing probability tables produced by the formulae in Table IV-2.

### 3. Non-normal Distributions with Equal and Unequal Ranges

The same sort of mean/median and probability analyses that have been performed for normal distributions can be performed for non-normal distributions. One must, however, first derive the formulae for the various curves and utilize the calculus to obtain the areas in questions and their shifting means and medians. The mathematics involved in this kind of analysis is more complex.

## APPENDIX C

## A SAMPLE CALCULATION FOR THE AGGREGATE MODEL

Here is a sample calculation of the median value of the social benefits for high school attainers and non-attainers.

Suppose that the attainment ratio stands at 30 percent. See Figure C-1. We know that the attainer group monopolizes the social benefits ranging in value from 0.52 to 3.9 standard deviations from the grand mean.

The median benefit for this group is thus $\mu_{\Phi} = 1.037\sigma$. This is the point under the $\Phi$ portion of the total distribution where half of the high school attainers (i.e., 15 percent) lie to the right and where the other half lie to the left.

The median social benefits for the remaining 70 percent of the total population (i.e., the non-attainer group) is $\mu_{\underline{\Phi}} = -0.385$ . This is the point under the $\underline{\Phi}$ portion of the total distribution where one half of the high school non-attainers (i.e., 35 percent) lie to the right and the other half lie to the left.

The median social benefit values are derived from the standardized normal distribution, which represents a particular normal distribution of social benefits. If it turns out that, for this particular normal distribution, the median of the total distribution is $8,000 with a standard deviation of $2,500, we can easily calculate the medians (in dollars) of the attainer and non-attainer groups.

Attainer Group Median: $10,593 = $8,000 + (1.037 x $2,500); non-Attainer Group Median: $7,038 = $8,000 + (-0.385 x $2,500).

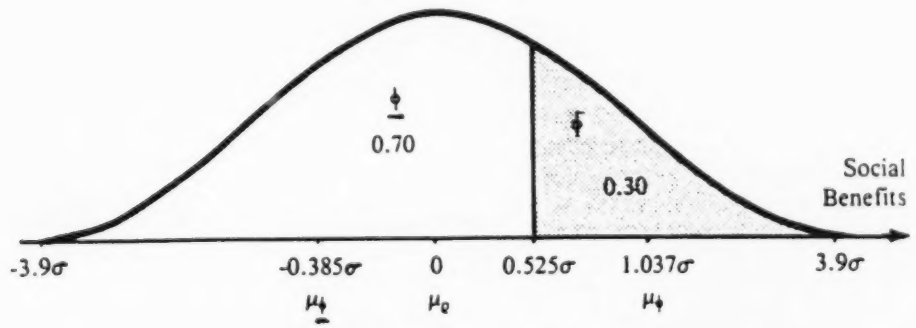


**Figure C-1. Standardized Normal Curve for the Distribution of Social Benefits**
($\Phi$ = high school attainment ratio; $\underline{\Phi}$ = non-attainment ratio; $\mu_{\rho} = 0$ = grand median, $\mu_{\Phi}$ = median social benefit for attainer group; $\mu_{\underline{\Phi}}$ = median social benefit for non-attainer group; $\sigma = 1$ = standard deviation)

It is probably unreasonable to apply the model at the lower attainment ratios where the power of the normative principle is very low. However, the model does serve to illustrate the idea that the relative benefit disparity between the two groups first decreases and then increases. This phenomenon suggests that a particular education policy appropriate for one stage of systemic growth might not be appropriate for another stage.

# APPENDIX D

## Empirical High School Attainment Data*

* National Center for Education Statistics (1994) Digest of Education Statistics. Washington, DC: U.S. Government Printing Office.

## Table 99.—High school graduates compared with population 17 years of age: 1869–70 to 1993–94

[Numbers in thousands]

| School year | Population 17 years old [1] | High school graduates | | | | | Graduates as a percent of 17-year-old population |
|---|---|---|---|---|---|---|---|
| | | Total [2] | Sex | | Control | | |
| | | | Male | Female | Public [3] | Private [4] | |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 1869–70 | 815 | 16 | 7 | 9 | — | — | 2.0 |
| 1879–80 | 946 | 24 | 11 | 13 | — | — | 2.5 |
| 1889–90 | 1,259 | 44 | 19 | 25 | 22 | 22 | 3.5 |
| 1899–1900 | 1,489 | 95 | 38 | 57 | 62 | 33 | 6.4 |
| 1909–10 | 1,786 | 156 | 64 | 93 | 111 | 45 | 8.8 |
| 1919–20 | 1,855 | 311 | 124 | 188 | 231 | 80 | 16.8 |
| 1929–30 | 2,296 | 667 | 300 | 367 | 592 | 75 | 29.0 |
| 1939–40 | 2,403 | 1,221 | 579 | 643 | 1,143 | 78 | 50.8 |
| 1947–48 | 2,261 | 1,190 | 563 | 627 | 1,073 | 117 | 52.6 |
| 1949–50 | 2,034 | 1,200 | 571 | 629 | 1,063 | 135 | 59.0 |
| 1951–52 | 2,086 | 1,197 | 569 | 627 | 1,056 | 141 | 57.4 |
| 1953–54 | 2,135 | 1,276 | 613 | 664 | 1,129 | 147 | 59.8 |
| 1955–56 | 2,242 | 1,415 | 680 | 735 | 1,252 | 163 | 63.1 |
| 1956–57 | 2,272 | 1,434 | 690 | 744 | 1,270 | 164 | 63.1 |
| 1957–58 | 2,325 | 1,506 | 725 | 781 | 1,332 | 174 | 64.8 |
| 1958–59 | 2,458 | 1,627 | 784 | 843 | 1,435 | 192 | 66.2 |
| 1959–60 | 2,672 | 1,858 | 895 | 963 | 1,627 | 231 | 69.5 |
| 1960–61 | 2,892 | 1,964 | 955 | 1,009 | 1,725 | 239 | 67.9 |
| 1961–62 | 2,768 | 1,918 | 938 | 980 | 1,678 | 240 | 69.3 |
| 1962–63 | 2,740 | 1,943 | 956 | 987 | 1,710 | 233 | 70.9 |
| 1963–64 | 2,978 | 2,283 | 1,120 | 1,163 | 2,008 | 275 | 76.7 |
| 1964–65 | 3,684 | 2,658 | 1,311 | 1,347 | 2,360 | 298 | 72.1 |
| 1965–66 | 3,489 | 2,665 | 1,323 | 1,342 | 2,367 | 298 | 76.4 |
| 1966–67 | 3,500 | 2,672 | 1,328 | 1,344 | 2,374 | 298 | 76.3 |
| 1967–68 | 3,532 | 2,695 | 1,338 | 1,357 | 2,395 | 300 | 76.3 |
| 1968–69 | 3,659 | 2,822 | 1,399 | 1,423 | 2,522 | 300 | 77.1 |
| 1969–70 | 3,757 | 2,889 | 1,430 | 1,459 | 2,589 | 300 | 76.9 |
| 1970–71 | 3,872 | 2,936 | 1,454 | 1,484 | 2,638 | 300 | 75.9 |
| 1971–72 | 3,973 | 3,002 | 1,487 | 1,515 | 2,700 | 302 | 75.6 |
| 1972–73 | 4,049 | 3,035 | 1,500 | 1,535 | 2,729 | 306 | 75.0 |
| 1973–74 | 4,132 | 3,073 | 1,512 | 1,561 | 2,763 | 310 | 74.4 |
| 1974–75 | 4,256 | 3,133 | 1,542 | 1,591 | 2,823 | 310 | 73.6 |
| 1975–76 | 4,272 | 3,148 | 1,552 | 1,596 | 2,837 | 311 | 73.7 |
| 1976–77 | 4,272 | 3,152 | 1,548 | 1,604 | 2,837 | 315 | 73.8 |
| 1977–78 | 4,286 | 3,127 | 1,531 | 1,596 | 2,825 | 302 | 73.0 |
| 1978–79 | 4,327 | 3,101 | 1,517 | 1,584 | 2,801 | 300 | 71.7 |
| 1979–80 | 4,262 | 3,043 | 1,491 | 1,552 | 2,748 | 295 | 71.4 |
| 1980–81 | 4,212 | 3,020 | 1,483 | 1,537 | 2,725 | 295 | 71.7 |
| 1981–82 | 4,134 | 2,995 | 1,471 | 1,524 | 2,705 | 290 | 72.4 |
| 1982–83 | 3,962 | 2,888 | 1,437 | 1,451 | 2,598 | 290 | 72.9 |
| 1983–84 | 3,784 | 2,767 | — | — | 2,495 | 272 | 73.1 |
| 1984–85 | 3,699 | 2,677 | — | — | 2,414 | 263 | 72.4 |
| 1985–86 | 3,670 | 2,643 | — | — | 2,383 | 260 | 72.0 |
| 1986–87 | 3,754 | 2,694 | — | — | 2,429 | 265 | 71.8 |
| 1987–88 | 3,849 | 2,773 | — | — | 2,500 | 273 | 72.1 |
| 1988–89 | 3,842 | 2,727 | — | — | 2,459 | 268 | 71.0 |
| 1989–90 | 3,574 | 2,588 | — | — | 2,320 | 268 | 72.4 |
| 1990–91 | 3,417 | 2,503 | — | — | 2,235 | 268 | 73.2 |
| 1991–92 | 3,381 | 2,471 | — | — | 2,212 | 259 | 73.1 |
| 1992–93 [5] | 3,430 | 2,512 | — | — | 2,255 | 257 | 73.2 |
| 1993–94 [5] | 3,440 | 2,513 | — | — | 2,255 | 258 | 73.1 |

[1] Derived from Current Population Reports, Series P-25. 17-year-old population adjusted to reflect October 17-year-old population.

[2] Includes graduates of public and private schools.

[3] Data for 1929–30 and preceding years are from *Statistics of Public High Schools* and exclude graduates of high schools which failed to report to the Office of Education.

[4] For most years, private school data have been estimated based on periodic private school surveys. For years through 1957–58, private includes data for subcollegiate departments of institutions of higher education and residential schools for exceptional children.

[5] Public high school graduates based on state estimates.

—Data not available.

NOTE.—Includes graduates of regular day school programs. Excludes graduates of other programs, when separately reported, and recipients of high school equivalency certificates. Some data have been revised from previously published figures. Because of rounding, details may not add to totals.

SOURCE: U.S. Department of Education, National Center for Education Statistics, *Statistics of Public High Schools; Biennial Survey of Education in the United States; Statistics of State School Systems; Statistics of Nonpublic Elementary and Secondary Schools*; Common Core of Data surveys; and U.S. Department of Commerce, Bureau of the Census, *Current Population Reports*, Series P-25. (This table was prepared April 1994.)



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